Class 11 Binomial Theorem

Binomial Theorem Identities

Here are some of the important identities which you must learn to ace this chapter-

$(x+y)^0= 1$

$(x+y)^1= x+y$

$(x+y)^2= x^2+2xy+y^2$

$(x+y)^3= x^3+y^3+3x^2y+3xy^2$

$(x+y)^4=(x+y)^3(x+y)=x^4+4x^{3-}y+4xy^3+6a^2b^24xy^3+y^4$

The illustrative expansion depicts that

- The given term number exceeds the index by 1.For example, in the expansion of $(x +y)^2$, term numbers are 3 whereas the index of $(x +y)^2$ is 2
- The powers of the first quantity 'x' go on decreasing by 1 whereas the powers of the second quantity 'y' keep on increasing by 1, in the terms that follow.
- In all the terms provided in the expansion, the addition of given indices of x and y is the equal and is called Binomial Theorem.

Class 11 Binomial Theorem: Important Concepts

Binomial theorem for any positive integer n,

$(x + y)^n = {}^nC_0a^n + {}^nC_1a^{n-1}b + {}^nC_2a^{n-2} b^2 + ...+ {}^nC_{n-1}a.b^{n-1} + {}^nC_nb^n$

Proof By applying mathematical induction principle the proof is obtained.

Let the given statement be

$P(n) : (x + y)^n = {}^nC_0a^n + {}^nC_1a^{n-1}b + {}^nC_2a^{n-2}b^2 + ...+ {}^nC_{n-1}a.b^{n-1} + {}^nC_nb^n$

For n = 1, we have

$P (1) : (x + y)^1 = {}^1C_0a^1 + {}^1C_1b^1 = x + y$ Thus, P (1) is true.

Suppose P (k) is true for some positive integer k, i.e.

$(x + y)^k = {}^kC_0a^k + {}^kC_1a^{k-1}b + {}^kC_2a^{k-2}b^2 + ...+ {}^kC_kb^k$ ... (1) We shall prove that P(k + 1) is also true, i.e.,

$(x + y)^{k+1} = {}^{k+1}C_0\, a^{k+1} + {}^{k+1}C_1\, a^kb + {}^{k+1}C_2\, a^{k-1}b^2 + ...+ {}^{k+1}C_{k+1}\, b^{k+1}$ Now, $(x + y)^{k+1} = (x + y)\,(x + y)^k$

$= (x + y)\,({}^kC_0\, a^k + {}^kC_1a^{k-1}\, y + {}^kC_2\, a^{k-2}\, b^2 +...+ {}^kC_{k-1}\, ab^{k-1} + {}^kC_k\, b^k)$ [from (1)]

$= {}^kC_0\, a^{k+1} + {}^kC_1\, a^kb + {}^kC_2a^{k-1}b^2 +...+ {}^kC_{k-1}\, a^2b^{k-1} + {}^kC_k\, ab^k + {}^kC_0\, a^kb$

$+ {}^kC_1a^{k-1}b^2 + {}^kC_2a^{k-2}b^3+...+ {}^kC_{k-1}ab^k + {}^kC_kb^{k+1}$[by actual multiplication]

$= {}^kC_0a^{k+1} + ({}^kC_1+ {}^kC_0)\, a^kb + ({}^kC_2 + {}^kC_1)a^{k-1}b^2 + ...$

$+ ({}^kC_k+ {}^kC_{k-1})\, ab^k + {}^kC_kb^{k+1}$ [grouping like terms] $= {}^{k+1}C_0a^{k+1} + {}^{k+1}C_1a^kb + {}^{k+1}C_2\, a^{k-1}b^2 +...+ {}^{k+1}C_kab^k + {}^{k+1}C_{k+1}\, b^{k+1}$

(by using ${}^{k+1}C_0=1$, ${}^kC_r + {}^kC_{r-1} = {}^{k+1}C_r$ and ${}^kC_k = 1= {}^{k+1}C_{k+1}$)

Thus, it has been proved that P (k + 1) is true whenever P(k) is true. .According to the principle of mathematical induction, P(n) is true for every positive integer n.

Properties of the Binomial Expansion

1. Total number of terms in the expansion of (x + a)n is (n + 1).
2. The sum of the indices of x and a in each term is n.
3. It is a correct expansion when the terms are complex numbers.
4. Terms that are equidistant from both ends will have coefficients that are equal. These are termed differently - binomial co-efficients.
5. General term in the expansion of (x + c)n is given by Tr + 1 = nCrx n – r ar .
6. It is important to note that the values first increase and then decrease as you go ahead in the expansion.
7. The coefficient of xr in the expansion of (1+ x)n is nCr.

Finding the Middle Term

Here are some simple formulas as derived from the class 11 binomial theorem chapter, that will help you understand the topic better.

Let's assume that we have an expression in the form $(a+b)^n$

And it has (n+1) terms, the middle term expansion of the $(a+b)^n$ depends on n.

When n is even such that n=2m and is a positive integer then the total number of terms will be 2m+1. Therefore, the middle term of the expression $(a+b)^n$ will be ½[(2m+1)+1]. Therefore, when n is even then the m+1 term and (n/2+1)th term will be the middle term.

When n is odd, such that n=2m+1 where m is a positive integer then the expansion of $(a+b)^n$ will have a total of (m+2) terms. The middle term of the expansion of $(a+b)^n$ will be (m+1)th and (m+2)th term or (n+1)/2 th term and (n+3)/2th term.

Important Terms on Binomial Theorem

- Binomial Expression: If an expression contains two terms combined by + or – is called a Binomial expression. For instance x+3, 2x-y etc.
- If the given expression is $(a+b)^n$ then in its expansion the coefficient of the first term will equal to the coefficient of last term. Likewise, the coefficient of the second last term is equal to the coefficient of second term. Hence, it is understandable that in the expansion of $(a+b)^n$ the terms from first term and the from the last term at equal distance are having the same coefficient
- The General term: Suppose there is an expression $(a+b)^n$ then the term(r+1)is called the general term for the expansion of the expression $(a+b)^n$. The general term is denoted by T_r+1

Class 11 Binomial Theorem: Solved Examples

Now that you are through with the chapter, here are some solved examples for you to practice-

Q1. Write down the approximation of $(0.99)^5$ by using the first three terms of its expansion.

We can write 99 as the sum or difference of two numbers having powers that are easier to calculate and then we can apply Binomial Theorem.

We can write it down in the form of 0.99= 1-0.01

Hence, $(0.99)^5 =(1-0.01)^5$

$= C(5, 0)\ (-0.01)^0 + C(5, 1)\ (-0.01)^1 + C(5, 2)\ (-0.01)^2 + \cdots$

$= 1 – 5(0.01) + 10(0.0001) + \cdots$

$\approx 1 – 0.05 + 0.001$

= 0.951

Example 1: By the use of Class 11 Binomial theorem, show that–$5n+6^n$ leaves behind on dividing by 25.

Solution:

For two numbers x and y if we can find numbers q and r such that x =yq + r, then we say that y divides x with q as quotient and r as remainder. Hence to logically explain ⁻ $5n+6^n$ gets a remainder of 1 when we divide it by 25, we are proving that– $5n+6^n = 1+25k$, where k is a natural number.

Thus we get,

$(1 + x)^n = {}^nC_0 + {}^nC_1a + {}^nC_2a^2 + ... + {}^nC_na^n$ For x = 5, we get

$(1 + 5)^n = {}^nC_0 + {}^nC_15 + {}^nC_25^2 + ... + {}^nC_n5^n$

i.e. $(6)^n = 1 + 5n + 5^2.{}^nC_2 + 5^3.{}^nC_3 + ... + 5^n$

i.e. $6^n – 5n = 1+5^2 ({}^nC_2 + {}^nC_35 + ... + 5^{n-2})$

or $6^n – 5n = 1+ 25 ({}^nC_2 + 5 .{}^nC_3 + ... + 5^{n-2})$

or $6^n – 5n = 25k+1$ where $k = {}^nC_2 + 5 .{}^nC_3 + ... + 5^{n–2}$

This shows that when divided by 25, 6^n – 5n leaves remainder 1

Example 2: The theorem could be illustrated through the expansion of $(x + 2)^6$

Solution:

$(x + 2)^6 = {}^6C_0x^6 + {}^6C_1x^5.2 + {}^6C_2x^42^2 + {}^6C_3x^3.2^3 + {}^6C_4x^2.2^4 + {}^6C_5x.2^5 + {}^6C_6.2^6$

$= x^6 + 12x^5 + 60x^4 + 160x^3 + 240x^2 + 192x + 64$

Thus $(x + 2)^6 = x^6 + 12x^5 + 60x^4 + 160x^3 + 240x^2 + 192x + 64$