

Permutation and Combination Class 11

What is Permutation?

Permutation refers to a particular arrangement of a set of objects in a defined order or a process of arranging numbers or letters in a sequence. We can represent permutations by:

- $P(n, k)$
- P_k^n
- ${}_n P_k$
- ${}^n P_k$
- $P_{n, k}$

Let us understand the term permutation better, consider the numbers 456, and 654. Both of them consist of the same digits 4, 5, 6. But, they are arranged in a different order. So, we can say they are different permutations of the digits 4, 5, 6. We can form many different permutations from a given set of objects taking all of the digits from the set at a time or a particular number of objects at a time. The number of permutations that can be formed is represented by r at a time out of n . We shall denote this number of permutations by $P(n, r)$. The picture given below contains the formula of permutation.

$$P(n, r) = \frac{n!}{(n - r)!}$$

Where, the symbol '!' denotes factorial, which is the product of all the integers less than or even equal to n , but should be greater or equal to 1.

For Example: How many license plates consisting of three different digits can be made out of given integers 3, 4, 5, 6, 7?

Solution:

It is just like arranging 3 objects out of 5 objects. So, we have

$$P(5,3) = 5! / (5-3)!$$

$$= 5 \times 4 \times 3 \times 2 \times 1 / 2 \times 1$$

$$= 5 \times 4 \times 3 = 60$$

Types of Permutation

You may across various types of permutations even in day to day life. Hence, as per permutation and combination class 11 chapter, permutation can be classified into different categories:

Permutation of Objects When Repetition Not Allowed

Permutations without repetitions are arrangements of n elements in different groups so that they can be differentiated in the order they have been placed.

For Example : Let us consider a set of alphabets: $A = \{ a, b, c, d, e \}$

Then, the permutation of these 5 alphabets can be arranged in the following ways: **abcde, dbeca, bedac, acbde, adcea, cdbae, edabc**, etc.

The permutation for a number of n elements is given by: $P_n = n! = n(n-1)(n-2).. 2.1$

The total number of elements in the above example is 5 and hence,

$$P_5 = 5! = 5.4.3.2.1 = 120$$

So, at least 60 permutations can be made with elements $A = \{ a, b, c, d, e \}$

Let us go through some more examples to be clear with this topic.

For Example: In how many different ways can the letters of the word “MEMBER” be arranged?Solution:

The word member consists of 6 letters, 2 of which are M's, 2 are E's and the rest are different.

So, writing $n = 6$, $p = 2$, and $q = 2$ the required number of permutations are: $n! / p!q!$

$$n! / p!q! = 6! / 2!2! = 180$$

Circular Permutation

According to the permutation and combination class 11 chapter, the permutations used when the objects/ elements are placed in a circular order. For Example, the number of circular permutations of “ n ” different things at all times is taken as **$P = (n-1)!$**

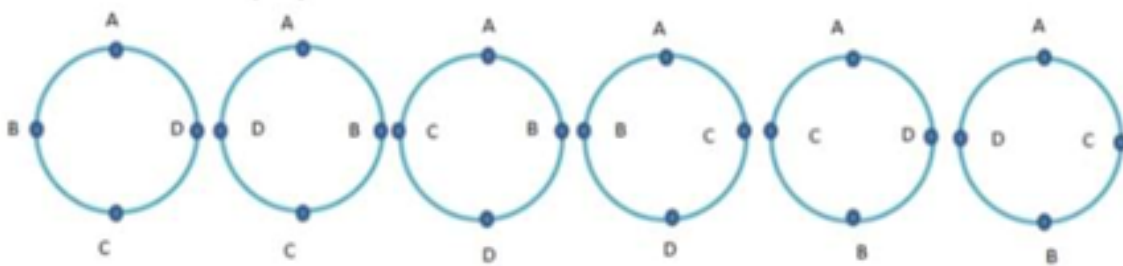
For Example: In how many different ways can the numbers on a clock face be arranged?

Solution:

In a clock face, there are 12 numbers. So they can be arranged in:

$(12-1)! = 11!$ ways

Going clockwise they are ABCD, ABDC, ADBC, ADCB, ACBD, ACDB. The arrangements BCDA, CDAB, DABC, are the same as the arrangement ABCD.



Permutation of Repeated Objects

To find the permutation of n , which is occurring any number of times or repetition of n elements, suppose there are r places, and there are n objects. The first place is filled up by any of the n objects. So, as mentioned in the permutation and combination class 11 chapter, there are ' n ' choices for filling up the first place.

Let us consider this example that from a lock you have to choose 10 numbers (0,1,2..9) and you picked 3 of them.

So, $10 * 10 * (3 \text{ times}) = 1000$ Permutations

The permutation of repetition is denoted by $PR(n,k)$

From a given set on n numbers, the numbers are formed, so that there are n_1 identical numbers of type 1, n_2 identical numbers for type 2.... and n_k numbers of type k . **Generally, repetitions are done by dividing permutation by the fractional numbers that are identical.**

Let us clear this out through an example, how many three-digit numbers can be formed from the numbers **0,1,2,3,4,5**

Solution:

Here, $n = 3$ and $K = 6$

First, the numbers will be separated into two blocks. The first set of blocks will consist of only one from the 5 digits because a number will not begin from zero.

So, $n=5$ and $K = 1$

The second block will consist of two numbers of any digit

$N=6$ and $K= 2$

$$P(5,1) \times Pr(6,2) = 5 \times 6^2 = 180$$

Properties Of Permutations

The permutation and combination class 11 chapter emphasis on a variety of properties, understanding which you will be through with an essential part of the concept.

$$(i) \quad {}^n P_n = n(n-1)(n-2) \dots 3 \times 2 \times 1 = n!$$

$$(ii) \quad {}^n P_0 = \frac{n!}{n!} = 1$$

$$(iii) \quad {}^n P_1 = n$$

$$(iv) \quad {}^n P_{n-1} = n!$$

$$(v) \quad {}^n P_r = n \cdot {}^{n-1} P_{r-1} = n(n-1) {}^{n-2} P_{r-2}$$

$$(vi) \quad {}^{n-1} P_r + r \cdot {}^{n-1} P_{r-1} = {}^n P_r$$

$$(vii) \quad \frac{{}^n P_r}{{}^n P_{r-1}} = n - r + 1$$

Principle of Multiplication

The principle states "If one operation can be done in m ways, there are also n ways to perform a second operation as well, then the number of ways to perform two operations together is $m \cdot n$ "

Principle of Addition

In an event of A and B , if the events do not occur simultaneously, i.e., first event A occurs in m different ways and the second event B occurs in n different ways, then one and the other will occur in $(m+n)$ ways. That is A will occur in $m+n$ ways and B will also occur in $m+n$ ways, according to permutations and combinations class 11 topic.

Combinations

A combination is known as selecting items from a combination or selection of all or part of a set of objects, without giving regard to the order in which the objects are selected. To calculate combination, we will use the formula:

$${}_n C_r = \binom{n}{r} = \frac{{}_n P_r}{r!} = \frac{n!}{r!(n-r)!}$$

Where, **n** represents the total number of items, and **r** represents the number of items being chosen at a time. An example of combination form class 11 NCERT book is given below-

Example: Calculate the value of C(12,9).

Solution:

$$C(12,9) = C(12,12-9)$$

$$= C(12,3) = 12!/9!3!$$

$$= 10 \cdot 11 \cdot 12 / 1 \cdot 2 \cdot 3$$

$$= 220$$

Difference Between Permutation And Combination

Permutation	Combination
A selection of r objects from a set of n objects in which the order of the selection matters.	The number of possible combinations of r objects from a set of n objects where the order of selection doesn't matter.
Permutation is used for lists (order matters).	Combination is used for groups (order doesn't matter).
It denotes the arrangement of objects.	It does not denote the arrangement of objects.
We can derive multiple permutations from a single combination.	Only a single combination can be derived from a single permutation.

They are defined as ordered elements.

They are defined as unordered sets.